Geometry 4-5 Study Guide: SSS & SAS (pp 250-257)

Attendance Problems

1. Name the angle formed by $\overrightarrow{AB}$ & $\overrightarrow{AC}$.

2. Name the the three sides of $\triangle ABC$.

3. $\triangle QRS \cong \triangle LMN$. Name all pairs of congruent corresponding parts.

- I can apply SSS and SAS to construct triangles and solve problems.
- I can prove triangles congruent by using SSS and SAS.

Vocabulary	
triangle rigidity	included angle

Common Core

CC.9-12.G.CO.8 Explain how the cri- teria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
CC.9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
CC.9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures.

In Lesson 4-3, you proved triangles congruent by showing that all six pairs of corresponding parts were congruent.
The property of **triangle rigidity** gives you a shortcut for proving two triangles congruent. It states that if the side lengths of a triangle are given, the triangle can have only one shape.
For example, you only need to know that two triangles have three pairs of congruent corresponding sides. This can be expressed as the following postulate.

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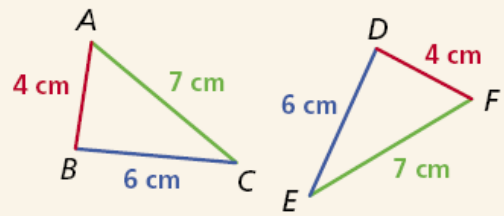
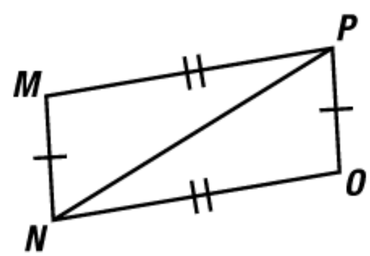
Postulate 4-4-1 **Side-Side-Side (SSS) Congruence**

POSTULATE	HYPOTHESIS	CONCLUSION
If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle FDE$

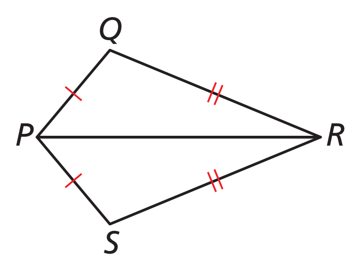
Remember!

Adjacent triangles share a side, so you can apply the Reflexive Property to get a pair of congruent parts.

<u>Video Example 1</u>. Use SSS to explain why $\triangle MNP \cong \triangle OPN$.



1 Using SSS to Prove Triangle Congruence



Use SSS to explain why $\triangle PQR \cong \triangle PSR$.

It is given that $\overline{PQ} \cong \overline{PS}$ and that $\overline{QR} \cong \overline{SR}$. By the Reflexive Property of Congruence, $\overline{PR} \cong \overline{PR}$. Therefore $\triangle PQR \cong \triangle PSR$ by SSS.

"Apples rule. They keep doctors away, and gain you sway with teachers."

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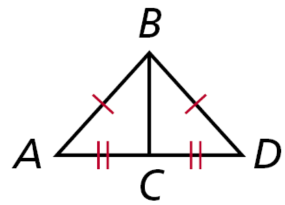
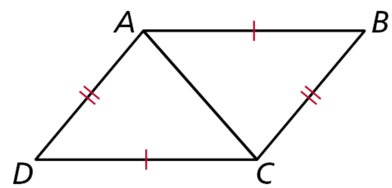
<u>Example 1</u>. Use SSS to explain why $\Delta ABC \cong \Delta DBC$.

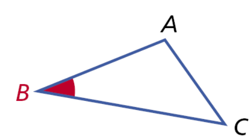


4. **<u>Guided Practice</u>.** Use SSS to explain why ΔABC ≅ ΔCDA.



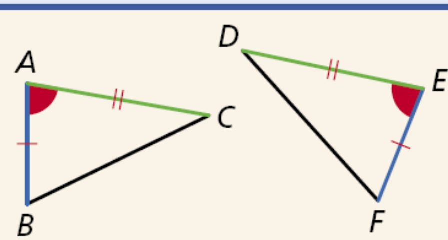
An **<u>included angle</u>** is an angle formed by two adjacent sides of a polygon.

$\measuredangle B$ is the included angle between sides $\overline{AB}$ and $\overline{BC}$.



It can also be shown that only two pairs of congruent corresponding sides are needed to prove the congruence of two triangles if the included angles are also congruent.

Postulate 4-4-2 Side-Angle-Side (SAS) Congruence

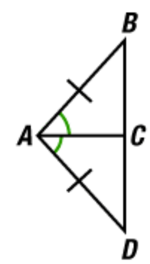
POSTULATE	HYPOTHESIS	CONCLUSION
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.		$\triangle ABC \cong \triangle EFD$

"Apples are fine, but I find today's teacher prefers a nice latte."

Caution

The letters SAS are written in that order because the congruent angles must be between pairs of congruent corresponding sides.

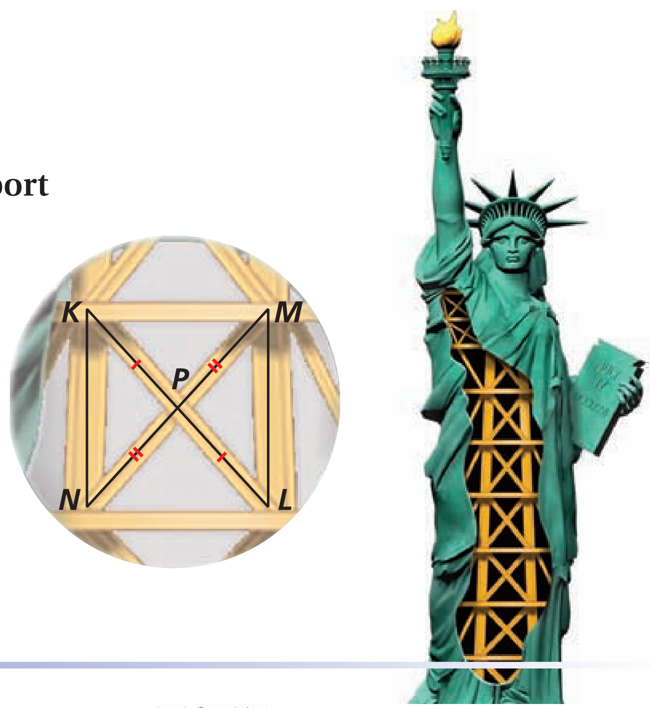
Video Example 2. Use the SAS postulate to prove $\triangle ACB \cong \triangle ACD$.



2 *Engineering Application*

The figure shows part of the support structure of the Statue of Liberty. Use SAS to explain why $\triangle KPN \cong \triangle LPM$.

It is given that $\overline{KP} \cong \overline{LP}$ and that $\overline{NP} \cong \overline{MP}$. By the Vertical Angles Theorem, $\angle KPN \cong \angle LPM$. Therefore $\triangle KPN \cong \triangle LPM$ by SAS.



"Is this *right*? The class list and the troublemaker list is the *same list*?!"

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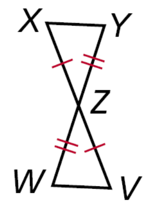
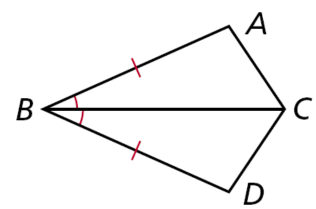
Example 2. The diagram shows part of the support structure for a tower. Use SAS to explain why $\Delta XYZ \cong \Delta VWZ$.



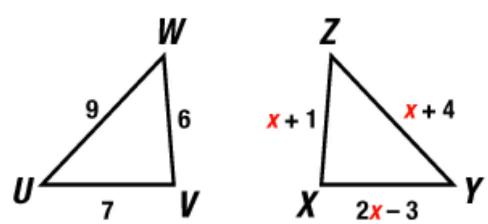
5. Guided Practice. Use SAS to explain why $\Delta ABC \cong \Delta DBC$.



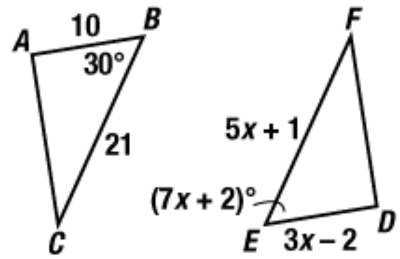
The SAS Postulate guarantees that if you are given the lengths of two sides and the measure of the included angles, you can construct one and only one triangle.

Video Example 3. Show that the triangles are congruent for the given value of the variable.

A. $\Delta UVW \cong \Delta YXZ$ when x = 5.



B. $\triangle ABC \cong \triangle DEF$ when x = 4.

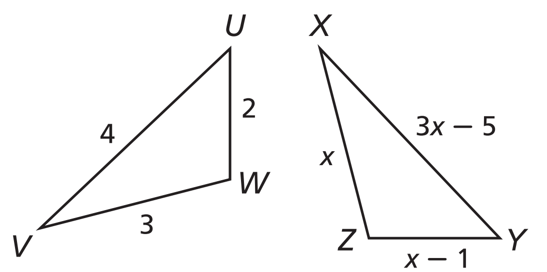


3 Verifying Triangle Congruence

Show that the triangles are congruent for the given value of the variable.

A $\triangle UVW \cong \triangle YXZ, x = 3$

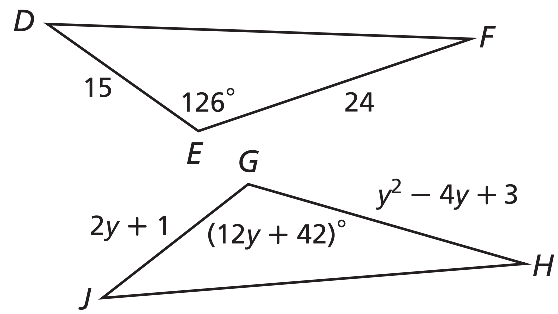
$ZY = x - 1$
$= 3 - 1 = 2$
$XZ = x = 3$
$XY = 3x - 5$
$= 3(3) - 5 = 4$



$\overline{UV} \cong \overline{YX}$. $\overline{VW} \cong \overline{XZ}$, and $\overline{UW} \cong \overline{YZ}$.
So $\triangle UVW \cong \triangle YXZ$ by SSS.

B $\triangle DEF \cong \triangle JGH, y = 7$

$JG = 2y + 1$
$= 2(7) + 1$
$= 15$
$GH = y^2 - 4y + 3$
$= (7)^2 - 4(7) + 3$
$= 24$
$m\angle G = 12y + 42$
$= 12(7) + 42$
$= 126°$



$\overline{DE} \cong \overline{JG}$. $\overline{EF} \cong \overline{GH}$, and $\angle E \cong \angle G$.
So $\triangle DEF \cong \triangle JGH$ by SAS.

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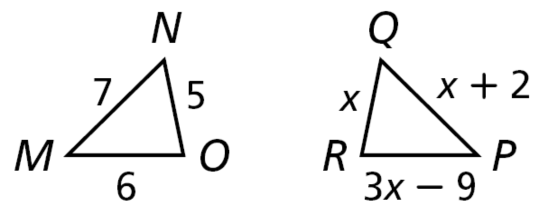
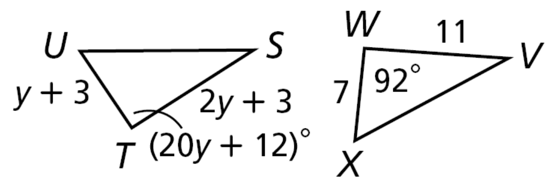
<u>Example 3</u>. Show that the triangles are congruent for the given value of the variable.

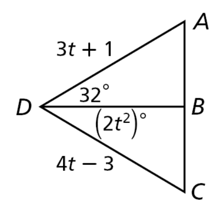
A. $\Delta MNO \cong \Delta PQR$, when $x = 5$.



B. $\Delta STU \cong \Delta VWX$, when $y = 4$.



6. <u>Guided Practice</u>. Show that $\Delta ADB \cong \Delta CDB$ if t = 4.



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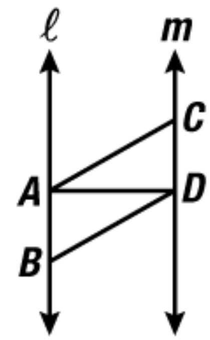
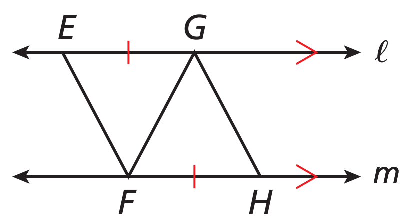
Video Example 4.

Given: $\ell \parallel m$
$\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ADB \cong \triangle ADC$

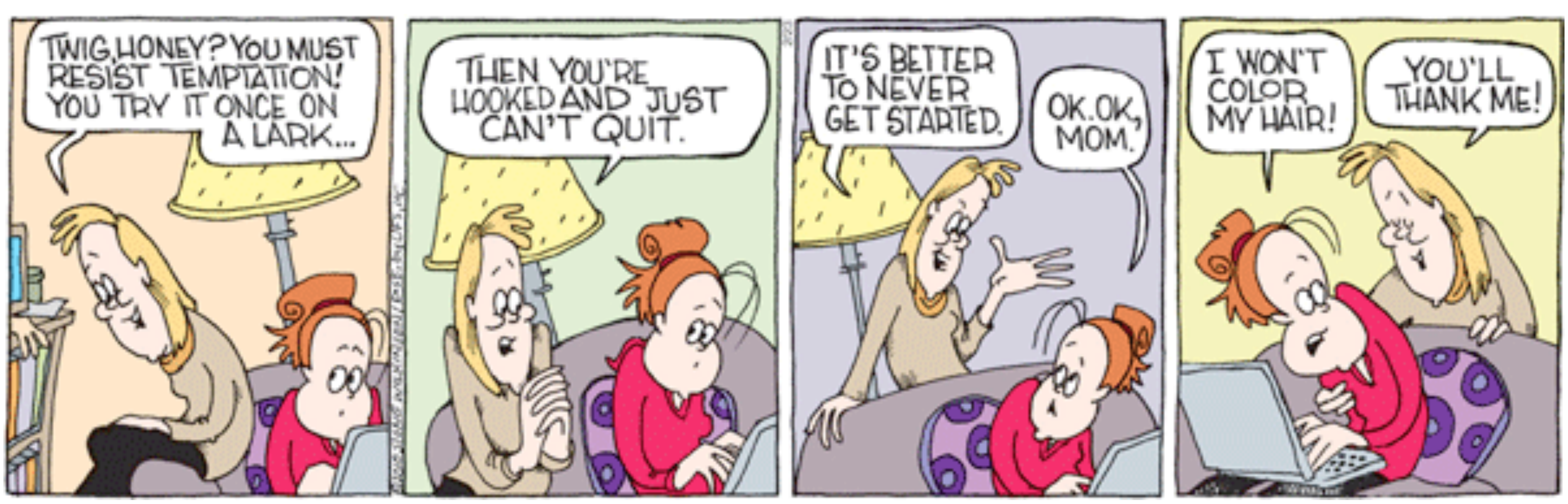


4 Proving Triangles Congruent



Given: $\ell \parallel m$, $\overline{EG} \cong \overline{HF}$
Prove: $\triangle EGF \cong \triangle HFG$
Proof:

Statements	Reasons
1. $\overline{EG} \cong \overline{HF}$	**1.** Given
2. $\ell \parallel m$	**2.** Given
3. $\angle EGF \cong \angle HFG$	**3.** Alt. Int. ∠s Thm.
4. $\overline{FG} \cong \overline{GF}$	**4.** Reflex Prop. of ≅
5. $\triangle EGF \cong \triangle HFG$	**5.** SAS *Steps 1, 3, 4*



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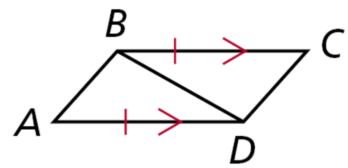
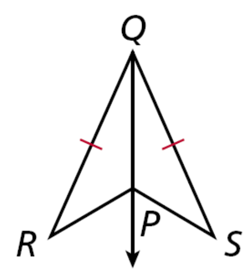
<u>Example 4</u>.



Given: $\overline{BC} \parallel \overline{AD}$
$\overline{BC} \cong \overline{AD}$

Prove: $\triangle ABD \cong \triangle CDB$

7. <u>Guided Practice</u>.



Given: $\overrightarrow{QP}$ *bisects* $\measuredangle RQS$.
$\overline{QR} \cong \overline{QS}$

Prove: $\triangle RQP \cong \triangle SQP$

4-5 Assignment: *(pp 255-256)* 8, 10, 12-14, 25.